LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2011

MT 1812 - ORDINARY DIFFERENTIAL EQUATIONS

Date : 05-11-2011 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

Answer all questions. Each question carries 20 marks.

1. (a) Prove that $x(t) = x_p(t) + x_h(t)$ is the general solution of $L(x(t)) = d(t)$ on I where $x_p(t)$ is any particular solution of $L(x(t)) = d(t)$ and $x_h(t)$ is the general solution of the homogeneous equation $L(x(t)) = 0$. (5)

(OR)

(b) Prove that $x = ct^2 + t + 3$, $t \geq 0$, is a solution of $t^2x'' - 2tx' + 2x = 6$. (5)

(c) Prove that $uL(v) - vL(u) = a_0(t)\frac{d}{dt}W[u,v] + a_1(t)W[u,v]$, where u, v are twice differentiable functions and a_0, a_1 are continuous on I. Also deduce Abel's formula. (15)

(OR)

(d) By the method of variation of parameters, find the general solution of $x'''(t) - x'(t) = cost$. (15)

2. (a) i) Find the indicial equation of $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$. (5)

(OR)

(b) Whenever n is an integer, positive or negative, show that $J_{-n}(X) = (-1)^n J_n(X)$. (5)

(c) Solve by Frobenius method, $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$. (15)

(OR)

(d) Solve the Legendre's equation, $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + l(l+1)y = 0$. (15)

3. (a) Show that the explicit expression for the legendre polynomials (1) $P_l(-1) = (-1)^l$ and (2) $P_l'(1) = \frac{1}{2}l(l+1)$. (5)

(OR)

(b) Show that F (1; p; p; x) = 1/ (1 – x). (5)

(c) State and prove Rodriguez's Formula and find the value of $\{8P_4(x) + 20P_2(x) + 7P_0(x)\}$. (15)

(OR)

(d) Solve the Bessel's equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ (15)

4. (a) Prove that all the eigen values of Strum - Liouville problem are real. (5)

(OR)

(b) Find the eigen values and eigen functions of $x'' + \lambda x = 0, x(0) = 0, x'(\pi) = 0$. (5)

(c) State and prove Picard's theorem for initial value problem. (15)

(OR)

(d) State Green's function. Prove that $x(t)$ is a solution of $L(x(t)) + f(t) = 0$, $a \le t \le b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$. (15)

5. (a) Prove that the null solution of the system $x' = A(t)x$ is asymptotically stable if and only if $|\phi(t)| \to 0$ as $t \to \infty$. (5)

(OR)

(b) Let a function $V(t, x)$ exist such that $V(t, 0) = 0$ for $t \in I$, $V(t, x)$ is bounded, first order partial derivatives of V with respect to $x_i (i = 1, 2 \dots n)$ are continuous on $I \times S_\rho$, $V(t, x)$is positive definite and $\dot{V}(t,x) \le 0$. Prove that $x' = f(t,x),\ t \ge t_0 \ge 0$ is stable. (5)

(c) Discuss the stability of autonomous systems. (15)

(OR)

(d) By Lyapunov direct method, discuss the stability of $x' = Ax$. (15)

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